

MSGUT : from Futility to Precision ^a

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We computed the complete gauge and chiral superheavy spectra and couplings in the Minimal Susy GUT and therefrom evaluated the dependence of 1-loop threshold corrections to the Weinberg angle and Unification scale as functions of the single complex parameter ξ that controls MSGUT symmetry breaking. The corrections are generically within 10% showing that contrary to longstanding conjectures, *high precision calculations are not futile but necessary and feasible in the $SO(10)$ MSGUT*. Effective superpotentials for $B - L$ violation and mass formulae of the MSSM matter supermultiplets including neutrino Type I and II seesaws were derived.

The MSGUT i.e the Supersymmetric $SO(10)$ GUT based on the **126**, **$\overline{126}$** , **210** Higgs multiplets^{1,2} is now a focus of multifaceted interest motivated by its economy and predictivity. This is the most natural and minimal renormalizable model^{3,4} compatible with the observed fermion spectra and “minimax” mixing^{5,6,7}. In such models the gauge coupling becomes strong above the perturbative unification scale $M_X \sim 10^{16} \text{ GeV}$ leading ineluctably^{8,9} to dynamical symmetry breaking of the GUT symmetry at a scale Λ_U (just above M_X) which is *calculably*^{9,18} determined by only the low energy data and structural features of the theory. UV gauge strength leads to a novel picture of elementarity and dual unification characterized by a new fundamental length scale $\sim \Lambda_U^{-1}$ characterizing the “hearts of quarks.”^{8,9}. We have calculated¹⁰ the complete mass spectrum and couplings of the MSGUT using the techniques developed by us¹² and therefrom the threshold corrections to $\sin^2 \theta_W$ and the perturbative unification scale. Contrary to longstanding conjectures¹³ threshold corrections at and to M_X are generically small although exceptional parameter regions exist. Other recent calculations of the same mass spectra using a different¹¹ method can be found in^{14,15}. Our spectra coincide (upto conventions) with those of the first reference above and we comment briefly on differences with the second reference at the end. See the main paper¹⁰ for detailed comments.

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We study a renormalizable globally supersymmetric $SO(10)$ GUT whose chiral supermultiplets consist of “AM type” totally antisymmetric tensors : $\mathbf{210}(\Phi_{ijkl})$, $\overline{\mathbf{126}}(\overline{\Sigma}_{ijklm})$, $\mathbf{126}(\Sigma_{ijklm})$ ($i, j = 1 \dots 10$) which break the GUT symmetry to the MSSM, together with Fermion mass (FM) Higgs $\mathbf{10}$ -plet (\mathbf{H}_i). The $\overline{\mathbf{126}}$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion neutrino masses and mixings (via the Type I and/or Type II mechanisms); three $\mathbf{16}$ -plets Ψ_A ($A = 1, 2, 3$) contain the matter including the three conjugate neutrinos ($\bar{\nu}_L^A$).

The superpotential is

$$\begin{aligned} W = & \frac{1}{2} M_H H_i^2 + \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klmn} \Phi_{mnij} + \frac{M}{5!} \Sigma_{ijklm} \overline{\Sigma}_{ijklm} \\ & + \frac{\eta}{4!} \Phi_{ijkl} \Sigma_{ijmno} \overline{\Sigma}_{klmno} + \frac{1}{4!} H_i \Phi_{jklm} (\gamma \Sigma_{ijklm} + \bar{\gamma} \overline{\Sigma}_{ijklm}) \\ & + h'_{AB} \psi_A^T C_2^{(5)} \gamma_i \psi_B H_i + \frac{1}{5!} f'_{AB} \psi_A^T C_2^{(5)} \gamma_{i_1} \dots \gamma_{i_5} \psi_B \overline{\Sigma}_{i_1 \dots i_5} \end{aligned} \quad (1)$$

In all the MSGUT has exactly 26 non-soft parameters⁴. The MSSM also has 26 non-soft couplings so the 15 parameters of W_{FM} must be essentially responsible for the 22 parameters describing fermion masses and mixings in the MSSM.

The GUT scale vevs that break the gauge symmetry down to the SM symmetry are^{1,2} $\langle (15, 1, 1) \rangle_{210} : \langle \phi_{abcd} \rangle = \frac{a}{2} \epsilon_{abcdef} \epsilon_{ef}$, $\langle (15, 1, 3) \rangle_{210} : \langle \phi_{ab\tilde{\alpha}\tilde{\beta}} \rangle = \omega \epsilon_{ab} \epsilon_{\tilde{\alpha}\tilde{\beta}}$, $\langle (1, 1, 1) \rangle_{210} : \langle \phi_{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}\tilde{\delta}} \rangle = p \epsilon_{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}\tilde{\delta}}$, $\langle \overline{\Sigma}_{\hat{1}\hat{3}\hat{5}\hat{8}\hat{0}} \rangle = \bar{\sigma}$, $\langle \Sigma_{\hat{2}\hat{4}\hat{6}\hat{7}\hat{9}} \rangle = \sigma$. The vanishing of the D-terms of the $SO(10)$ gauge sector potential imposes only the condition $|\sigma| = |\bar{\sigma}|$. Except for the simpler cases corresponding to enhanced unbroken symmetry ($SU(5) \times U(1)$, $SU(5)$, $G_{3,2,2,B-L}$, $G_{3,2,R,B-L}$ etc)^{4,14} this system of equations is essentially cubic and can be reduced to the single equation⁴ for a variable $x = -\lambda\omega/m$, in terms of which the vevs $a, \omega, p, \sigma, \bar{\sigma}$ are specified :

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2 \quad (2)$$

where $\xi = \frac{\lambda M}{\eta m}$. This exhibits the crucial importance of the parameter ξ .

Using the above vevs and the methods of¹² we calculated the complete gauge and chiral multiplet GUT scale spectra and couplings for the 52 different MSSM multiplet sets falling into 26 different MSSM multiplet types of which 18 are unmixed while the other 8 types occur in multiple copies and mix via upto 5×5 matrices. (*On a lighter note* : the occurrence yet again of the ‘mystic’ String Theory number 26 demonstrates that one can do just as well without string theory !)

If the serendipity of the threefold gauge unification at M_X^0 is to survive closer examination the MSGUT must answer the query : *Are the one loop values of $\sin^2 \theta_W$ and M_X generically stable against superheavy threshold calculations ?* The parameter $\xi = \lambda M / \eta m$ is the most crucial determinant of the mass spectrum. The formulae for the threshold corrections are^{16,17}

$$\Delta^{(th)}(\log_{10} M_X) = .0217 + .0167(5\bar{b}'_1 + 3\bar{b}'_2 - 8\bar{b}'_3) \log_{10} \frac{M'}{M_X^0} \quad (3)$$

$$\Delta^{(th)}(\sin^2 \theta_W(M_S)) = .00004 - .00024(4\bar{b}'_1 - 9.6\bar{b}'_2 + 5.6\bar{b}'_3) \log_{10} \frac{M'}{M_X^0} \quad (4)$$

Where \bar{b}'_i are 1-loop beta function coefficients for multiplets with mass M' .

We plot these threshold corrections for a range of values of ξ keeping the other “insensitive” parameters fixed at randomly chosen representative values $\lambda = 0.12$; $\eta = 0.21$; $\gamma = 0.23$; $\bar{\gamma} = 0.35$.

Generically, effects on $\sin^2 \theta_W(M_S)$ are less than 10 % of the 1-loop values and the change in M_X is also not drastic. Near special points (among which one recognizes certain known points of

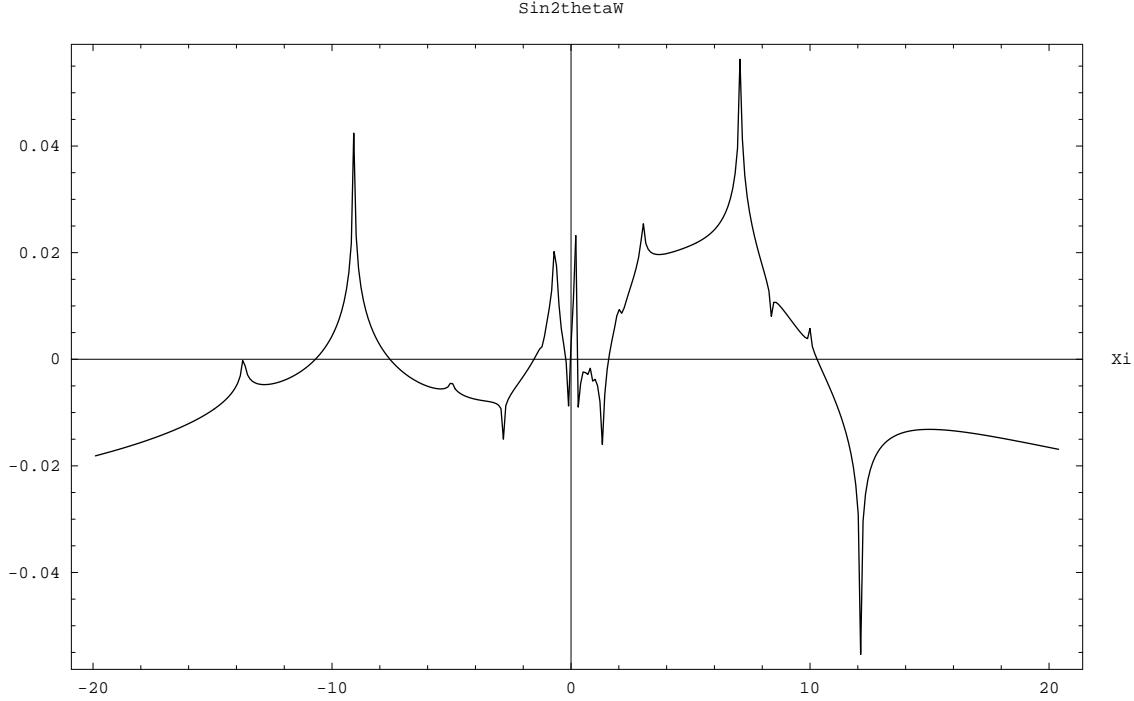


Figure 1: Plot of the threshold corrections to $\text{Sin}^2\theta_w$ vs ξ for real ξ : real solution for x .

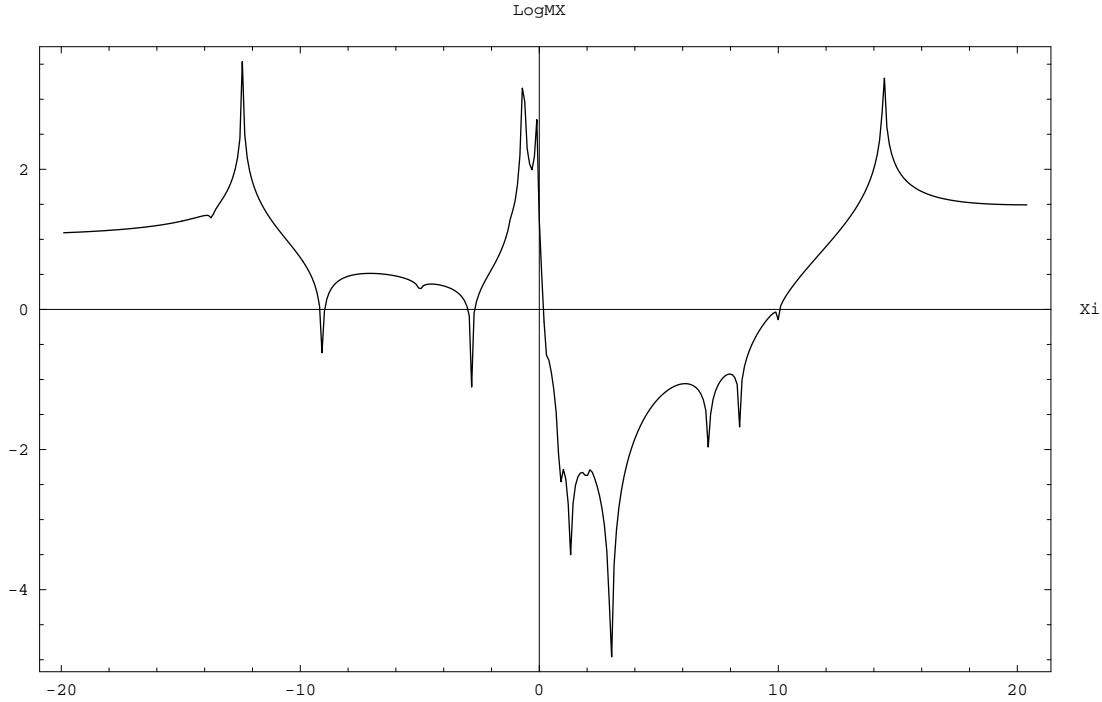


Figure 2: Plot of the threshold corrected $\text{Log}_{10}M_X/M_X^0$ vs ξ for real ξ : real solution for x .

enhanced symmetry^{4,14)} the corrections may be large but never explosively so. Regions where the threshold corrections to $\text{Log}_{10}M_X$ are large need special examination with regard to their phenomenological viability and consistency with the one scale breaking picture.

For complex values of ξ as shown we find changes are generically less than 10% for $\text{Sin}^2\theta_w$

while M_X changes by a factor of 10 or less. Thus our central point is that: *contrary to expectations in the literature*¹³, *precision RG analysis of the SO(10) MSGUT is far from being futile but rather is necessary for precision unification*, since the hierarchy of magnitudes between $O(\alpha^{-1})$ terms and 1-loop threshold/2-loop gauge coupling terms ($O(1)$ effects¹⁷) is generically maintained. However the mechanism that enforces the otherwise unreasonable insensitivity to strong growth of the coupling above M_X must be found¹⁸.

Using our methods we can compute all couplings of MSSM submultiplets in terms of GUT couplings and on integrating out the heavy triplet Higgs supermultipletst, \bar{t} one obtains:

$$W_{eff}^{\Delta B \neq 0} = L_{ABCD}(\frac{1}{2}\epsilon Q_A Q_B Q_C L_D) + R_{ABCD}(\epsilon \bar{e}_A \bar{u}_B \bar{u}_C \bar{d}_D)$$

Where, $L_{ABCD} = \mathcal{S}_1^1 h_{AB} h_{CD} + \mathcal{S}_1^2 h_{AB} f_{CD} + \mathcal{S}_2^1 f_{AB} h_{CD} + \mathcal{S}_2^2 f_{AB} f_{CD}$ and similarly for R_{ABCD} . Here $\mathcal{S} = \mathcal{T}^{-1}$ and $W = \bar{t} \mathcal{T} t + \dots$

Similarly we can calculate the mass matrices of matter supermultiplets, at the GUT scale including their dependence on the mixing coefficients defined by the fine tuning condition necessary to keep one pair of MSSM Higgs doublets light. See the main paper¹⁰.

Our method is different from the computer based method of^{3,14,15} and is more complete, especially regarding couplings. Precision calculations that ignore threshold effects in SO(10) GUTs seem to be of dubious validity. The authors of hep-ph/0405300 claimed that the mass spectra listed above were not consistent with the requirements of $SU(5)$ or $SU(5) \times U(1)$ symmetry at the special points where $p = a = \pm\omega$. *However this is entirely incorrect.* We have checked that in fact our mass terms naturally respect these symmetries fully at these special solutions since they organize into appropriate $SU(5)$ invariants when these conditions hold.

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